

TECHNICAL REPORT

POWER SPECTRUM ANALYSIS
OF
INTERNAL WAVES FROM
OPERATION STANDSTILL

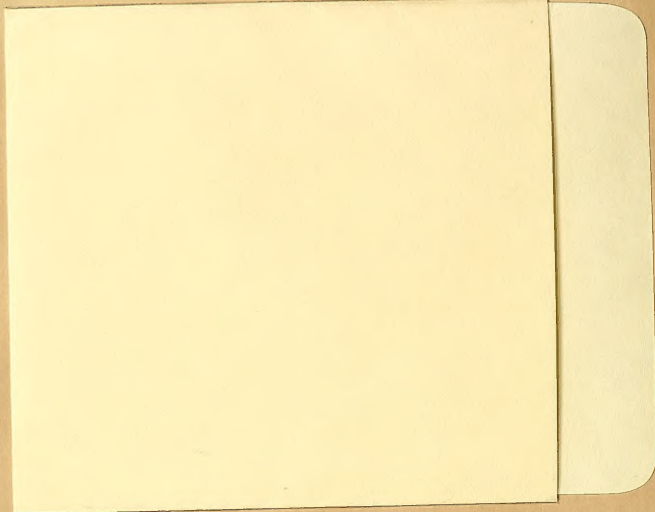
*Applied Oceanography Branch
Division of Oceanography*

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ABSTRACT

Temperature fluctuations in the ocean obtained from data collected during Operation STANDSTILL are analyzed by power spectrum techniques to determine the periods of internal waves. The analysis indicates that the power outside the noise level is concentrated in periods from 20 to 26 hours. Two physical causes for diurnal periods are suggested: eddies due to instability south of the Gulf Stream, and free oscillations, controlled by the Coriolis force, in resonance with the diurnal tidal components. It is concluded that the latter cause probably is responsible for the peak power of internal waves at periods between 22 and 24 hours.

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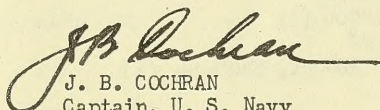
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FOREWORD

Many phases of naval operations involve the application of oceanographic principles. In order to predict changes in oceanographic parameters, it is necessary to understand the nature of the physical processes that are producing the changes. Accurate methods of predicting changes which occur in the thermal structure of the ocean cannot be obtained without consideration of internal waves.

This report presents the results of research concerning the existence and periods of internal waves in an area northeast of Bermuda. Activities receiving this publication are requested to forward their comments to the Hydrographic Office.


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Captain, U. S. Navy
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I. INTRODUCTION

In June and July 1953 Operation STANDSTILL was conducted in the area northeast of Bermuda. During the course of this operation an anchor station was maintained at $33^{\circ}33'N$, $62^{\circ}25'W$ for a period of 25 days, from 11 June to 6 July. Bathythermograph readings were taken every half hour as part of the oceanographic observations made at the anchor station. The following discussion deals with one aspect of these observations: the analysis of the wave spectrum of internal waves.

II. INTERNAL WAVES

Previous discussion of the theory and existence of internal waves in the ocean has been directed toward the demonstration of the occurrence of internal waves of tidal period. Thus, Haurwitz (1953) has examined the temperature data from Meteor stations 385 and 438 and the Altair anchor stations and has concluded that the oscillations at Meteor station 385 show possible existence of internal waves of semidiurnal tidal period.

With the recent development of the techniques of power spectrum analysis (Pierson, 1952), it is possible to analyze the spectral components of any suitable series of data comprising a Gaussian process. If it is assumed that the fluctuations of temperature in the ocean represent a stationary Gaussian process, the power spectrum analysis will indicate the significant periods present in the series and the power associated with these periods. In this way, it is possible to determine whether internal waves of tidal periods exist, as well as other periods greater and smaller.

III. ANALYSIS OF DATA

An examination of all the BT data showed that the deepest continuous isotherm was $66^{\circ}F$. Accordingly, the depth of this isotherm was determined from each half-hourly bathythermogram. These data are presented in table 1. Two ships in succession participated in this operation, the USS RE-HOBOTH and USS SAN PABLO. The frequency of the readings of the depth of the $66^{\circ}F$. isotherm is given for 25-foot intervals from 76 to 1,100 feet. This frequency distribution is graphed in figure 1.

Theoretically, distributions which are to be analyzed by power spectrum methods should be normal, i.e., Gaussian (Tukey, 1949). The present distribution has an upper limit at the surface and therefore is of the Pearson Type III. A normal distribution curve was fitted to the observed distribution, taking only the depths from 76 feet to 575 feet. The resulting curve is shown in figure 1 as a dashed line. The fitted curve indicates that the observed curve does not depart greatly from the normal distribution. Thus, since the frequency distribution is nearly normal, power spectrum analysis by the usual methods will not give meaningless results. The series of 1191 readings was analyzed for its power spectrum (Pierson and Marks, 1952) in five different ways:

TABLE 1. FREQUENCY OF OCCURRENCE OF 66° F. ISOTHERM
IN CLASS INTERVALS OF DEPTH

FREQUENCY

Depth ft.	USS REHOBOTH	USS SAN PABLO	Total Observed	Fitted Frequency (Unadjusted)
76-100	0	2	2	13
100-125	0	3	3	25
126-150	5	2	7	38
151-175	25	2	27	57
176-200	52	36	88	80
201-225	50	51	101	98
226-250	70	73	146	119
251-275	26	78	104	129
276-300	67	94	161	128
301-325	45	56	101	118
326-350	59	45	104	99
351-375	55	22	77	77
376-400	52	36	88	54
401-425	14	5	19	37
426-450	16	12	28	22
451-475	9	4	13	13
476-500	14	23	37	6
501-525	1	3	4	3
526-550	2	1	3	1
551-575	0	0	0	
576-600	0	14	14	
751-775	0	2	2	
776-800	0	6	6	
801-825	0	0	0	
826-850	0	1	1	
851-875	0	2	2	
876	0	4	4	
901-925	0	4	4	
926-950	0	4	4	
951-975	0	3	3	
976-1000	0	9	9	
1001-1025	0	4	4	
1026-1050	0	4	4	
1051-1075	0	1	1	
1076-1100	0	4	4	
Total	563	628	1191	1116

Note: Unadjusted total fitted frequency of 1116 compares with observed total of 1113 in range from 101 to 550 feet.

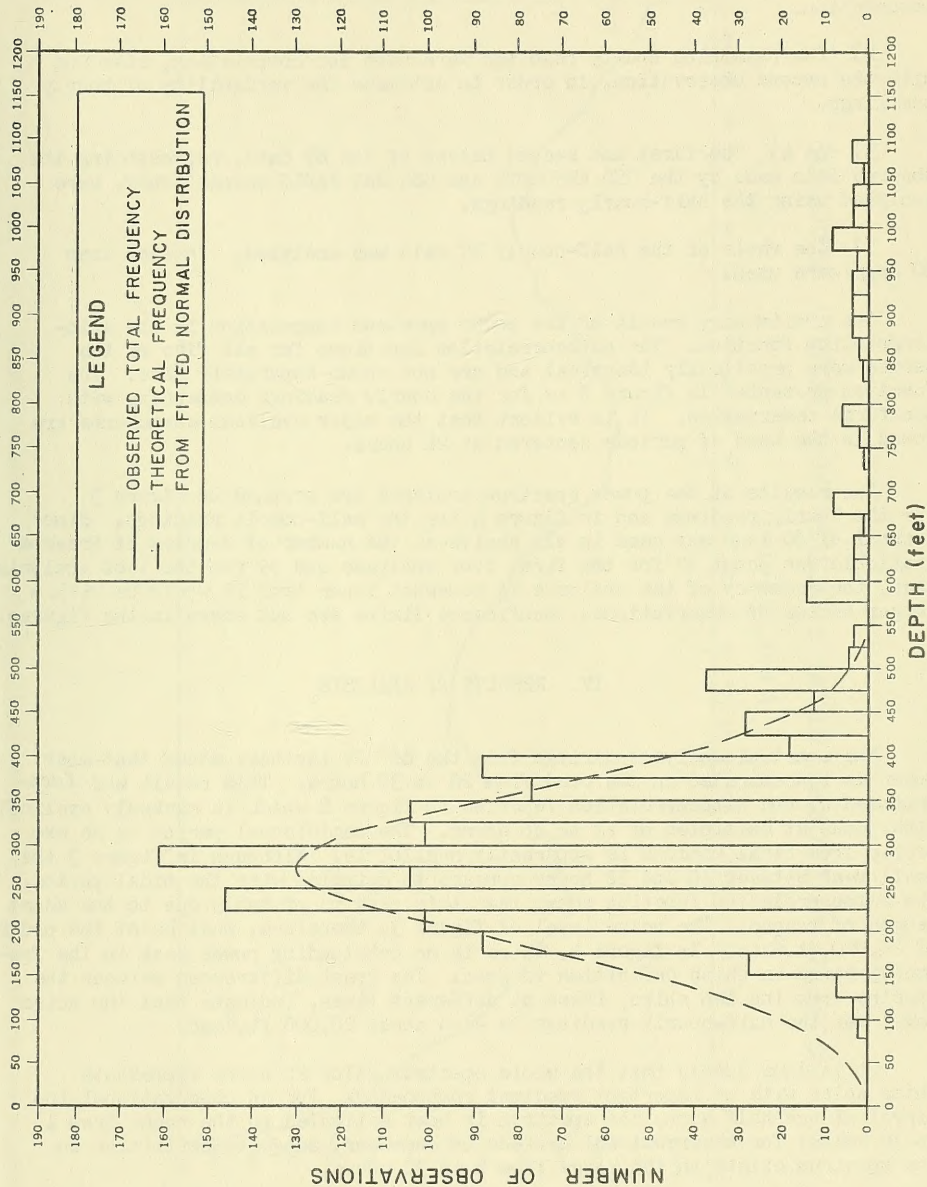


FIGURE 1. FREQUENCY DISTRIBUTION OF DEPTH OF 66°F. ISOTHERM

1) Hourly readings were used for computation, starting with the first observation.

2) The remaining hourly readings were used for computation, starting with the second observation, in order to estimate the variability of hourly samplings.

3) and 4) The first and second halves of the BT data, representing the observations made by the USS REHOBOTH and USS SAN PABLO respectively, were analyzed using the half-hourly readings.

5) The whole of the half-hourly BT data was analyzed. In each case 60 lags were used.

One preliminary result of the power spectrum computation is the autocorrelation function. The autocorrelation functions for all five of the series were practically identical and are not shown separately here. The function presented in figure 2 is for the hourly readings commencing with the first observation. It is evident that the major cyclical components are found in the band of periods centered at 24 hours.

The results of the power spectrum analyses are graphed in figure 3 for the hourly readings and in figure 4 for the half-hourly readings. Since a total of 60 lags was used in all analyses, the number of degrees of freedom available was about 19 for the first four analyses and 39 for the last analysis. Thus, the accuracy of the analyses is somewhat lower than it would be with a longer series of observations. Confidence limits are not shown in the figures.

IV. RESULTS OF ANALYSIS

The spectral analysis derived from the 66° F. isotherm shows that most power is concentrated in the band from 20 to 30 hours. This result was foreshadowed by the autocorrelation function in figure 2 which is markedly cyclical with peaks at multiples of 22 to 26 hours. The semidiurnal period to be expected from tidal motions is apparently negligible. Although in figure 3 the small peak between 10 and 12 hours appears to coincide with the tidal period, the autocorrelation function shows that this peak is probably due to the short length of record. The noise level of figure 3, therefore, must be of the order of 20,000 ft.²sec. In figure 4, there is no outstanding power peak in the frequency range in which definition is good. The great differences between the spectra from the two ships, taken at different times, indicate that the noise level for the half-hourly readings is also about 20,000 ft.²sec.

It is thus likely that the whole spectrum below 20 hours represents white noise with no important spectral components. For an observational interval of one-half hour, the spectrum is best estimated in the range from 1 to 10 hours; for observational periods of one hour, adequate definition in the spectrum exists in the range from 2 to 30 hours.

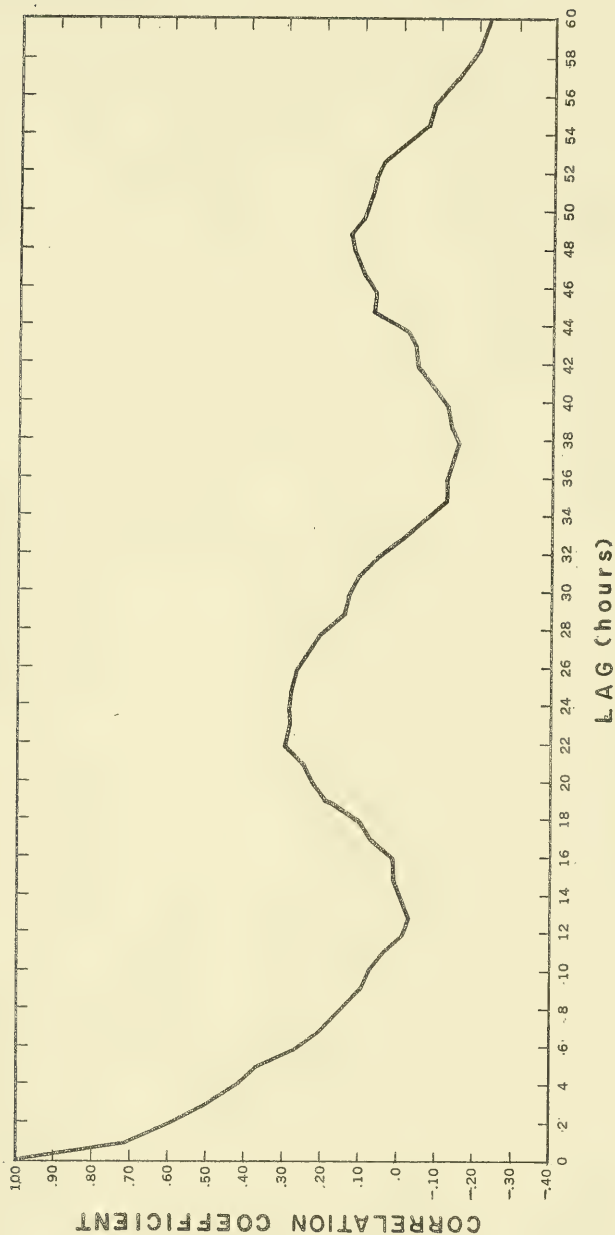


FIGURE 2. AUTOCORRELATION FUNCTION FOR HOURLY READINGS

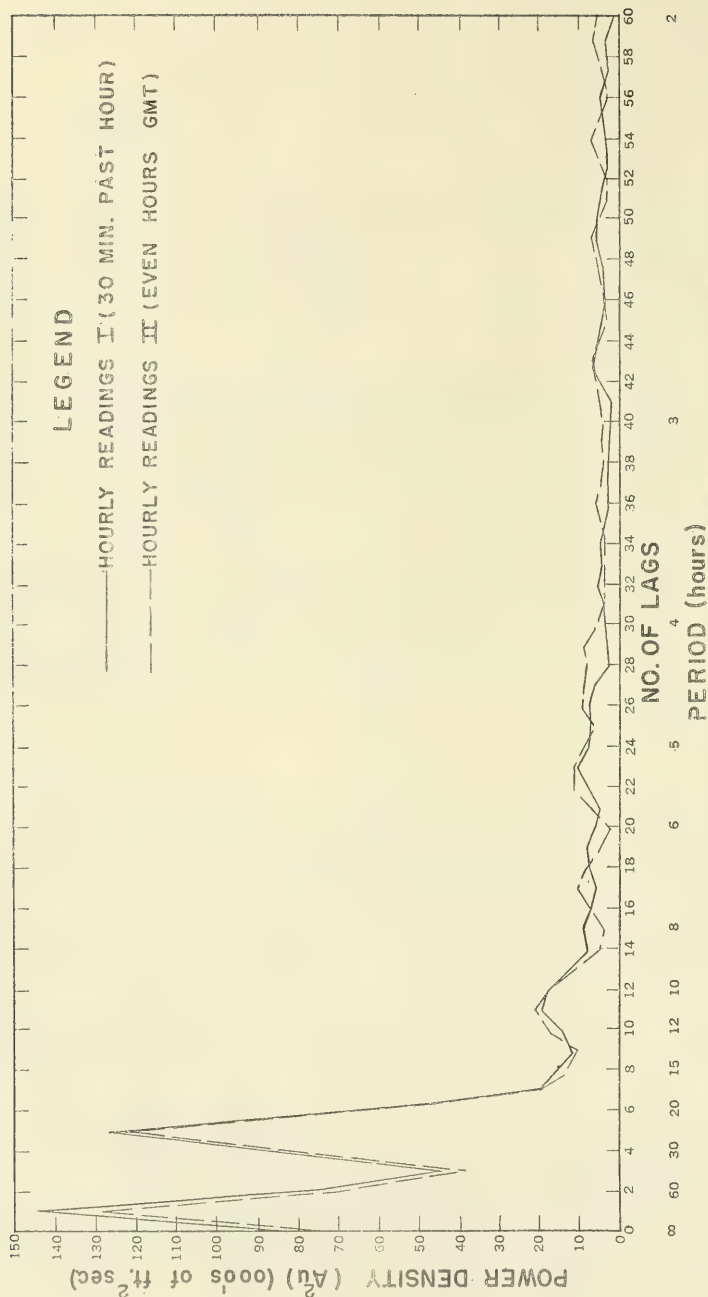


FIGURE 3. POWER DENSITY FUNCTIONS FOR HOURLY READINGS

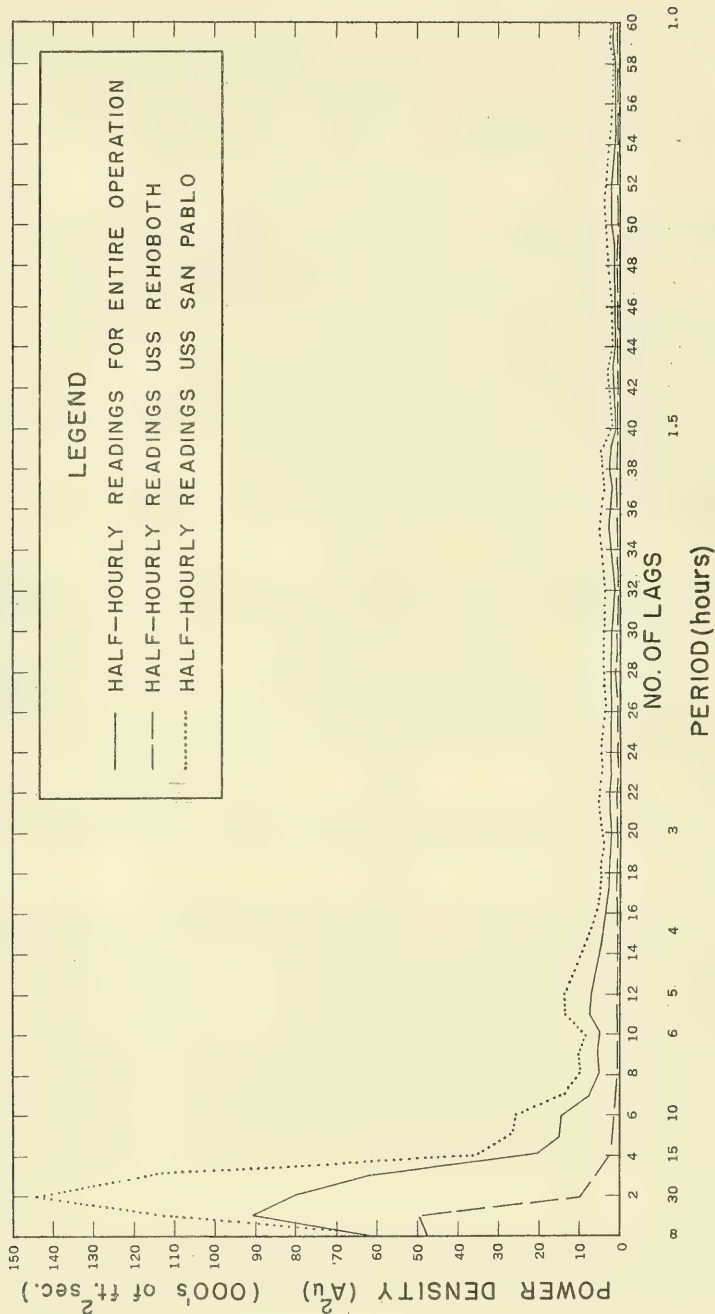


FIGURE 4. POWER DENSITY FUNCTIONS FOR HALF-HOURLY READINGS

In order to obtain a very accurate power spectrum analysis of waves of periods from 10 to 30 hours, which include the tidal periods, the time interval would have to be 3 to 5 hours and the length of the period, therefore, at least 3 to 5 times as long as the present records, i.e., from 75 to 125 days. Such a record could be obtained at a fixed weather ship location if regular bathythermograph readings were taken there.

The presence of aliasing in the record may be important. Pierson (1952) defines aliasing by saying that if the observational interval is too large, the power per band width can have other values of power from other frequencies aliased into (or added into) the true values for the particular band desired. If there is any large amount of power associated with periods of less than one hour in the present case, the spectral analysis is in error by that amount. Schule (1952) has discussed the existence of internal waves with periods of minutes. In order to show that internal waves with such periods have appreciable power, it would be necessary to have records taken every minute, comprising a large number of observations with lags of 200 to 500 taken into account. However, if the power associated with internal waves of periods less than one hour is low, i.e., in the noise level, it then becomes possible to consider that the spectrum is white below about 20 hours and that the conclusions of the preceding paragraphs apply.

The mean depth of the 66° F. isotherm was 324 feet and its standard deviation 87 feet. The mechanism which caused approximately diurnal periods in the isothermal depth could not have been vertical heating, for two reasons: the depth is too great, and the isotherm was located below the thermocline, which was fairly sharp. A horizontal pulsation of water of different temperatures could have caused the observed change. The average diurnal change in the depth of the isotherm was about 150 feet. It remains therefore to find the mechanism which caused diurnal advection of heat into the area of Operation STANDSTILL.

V. EDDY MOTION IN THE AREA OF OPERATION STANDSTILL

A. Eddies Resulting From Instability

As a result of a survey of the Gulf Stream made almost simultaneously with Operation STANDSTILL, it has been suggested that regular pulses in the Gulf Stream may be initiated by the diurnal tide of the Gulf of Mexico (Von Arx, Bumpus, and Richardson, 1954). At intervals of 24 hours the tidal motion may set into action jets of fast-moving water and produce noticeable fluctuations in the structure of the Gulf Stream. On the south side of the stream, it may be postulated that eddies, set up in phase with the inertial period, influenced the area of Operation STANDSTILL. Each eddy contained a jet of warmer water in swift motion followed by a slower moving current of cooler water. As this alternately warm and cool water passed through the area of the operation, the isotherms moved upward and downward in response to the water movement.

The regular pulses in the Gulf Stream may be compared mathematically to perturbations in a geostrophic flow. If a geostrophic flow is perturbed, the flow vector will show an acceleration which will cause a particle to deviate from geostrophic flow. This deviation may be represented as

$$\frac{d\vec{V}}{dt} = f(\vec{V} - \vec{V}_g) \times \vec{k} \quad (1)$$

where

f = the Coriolis parameter,

\vec{V} = the velocity vector,

\vec{V}_g = the geostrophic velocity vector, and

\vec{k} = the unit vector in an upward direction.

Substituting

$$\begin{aligned} \vec{V} &= \vec{V}_0 + \frac{d\vec{V}}{dt} dt \\ \vec{V}_g &= \vec{V}_{g0} + d\vec{V}_g = \vec{V}_{g0} + (d\vec{s} \cdot \vec{\nabla}) \vec{V}_{g0} \end{aligned}$$

since every velocity vector may be represented by the initial velocity plus an increment due to the acceleration, the resulting equation is

$$\frac{d\vec{V}}{dt} = f \left[\vec{V}_0 + \frac{d\vec{V}}{dt} dt - \vec{V}_{g0} - (d\vec{s} \cdot \vec{\nabla}) \vec{V}_{g0} \right] \times \vec{k} \quad (2)$$

Separating this equation into the components of velocity u and v gives

$$\frac{du}{dt} = f \left[V_0 - V_{g0} - \left(f + \frac{\partial V_{g0}}{\partial t} \right) dx - \frac{\partial V_{g0}}{\partial y} dy \right] = \vec{i} \cdot \frac{d\vec{V}}{dt} \quad (3)$$

$$\frac{dv}{dt} = f \left[\left(\frac{\partial u_{g0}}{\partial y} - f \right) dy + \frac{\partial u_{g0}}{\partial x} dx - (u_0 - u_{g0}) \right] = \vec{j} \cdot \frac{d\vec{V}}{dt} \quad (4)$$

where

dx = Perturbed displacement eastward, and

dy = Perturbed displacement northward.

In equations 3 and 4 the unit vectors \vec{i} and \vec{j} are directed toward the east and north respectively, so that the equations are the equations of motion in the two directions.

The increment of energy associated with the displacement may be represented as

$$dE = \frac{du}{dt} dx + \frac{dv}{dt} dy. \quad (5)$$

Substituting the values of $\frac{du}{dt}$ and $\frac{dv}{dt}$ from equations 3 and 4 gives

$$dE = f \left[\left(\frac{\partial u_0}{\partial y} - f \right) (dy)^2 - \left(\frac{\partial v_0}{\partial x} - f \right) (dx)^2 + \left(\frac{\partial u_0}{\partial x} - \frac{\partial v_0}{\partial y} \right) dy dx \right]. \quad (6)$$

This development is a special application to incompressible fluids of the more general exposition by Van Neighem (1951). Equation 6 contains the criterion for stability. Assuming flow from west to east (so that the only perturbations will be in the y-direction), the last two terms of the equation will drop out, and the stability depends on the sign of the first factor.

If $\frac{\partial u_0}{\partial y} - f > 0$ or $\frac{\partial u_0}{\partial y} > f$, the perturbed energy of the

system is increasing and the system is unstable; if $\frac{\partial u_0}{\partial y} < f$, the system is stable. Therefore, the equilibrium condition is attained when

$\frac{\partial u_0}{\partial y} = f = 2\omega \sin \phi$. Since the latitude of Operation STANDSTILL was $33^{\circ}33'N$, $\sin \phi$ is 0.553, and the equilibrium or inertial period is therefore 24 hours divided by 2 times 0.553, or 21.7 hours.

Equation 6 may be used to show that the maximum possible velocity gradient toward the right-hand side of a west-to-east flow must be smaller than the Coriolis Parameter (f) or else the flow becomes unstable and eddies form. On the left-hand side of the current, however, an increase in shear merely adds to the stability. Two illustrations show how the Gulf Stream system conforms to these principles. Figure 5, taken from Worthington (1955), is a velocity section across the Gulf Stream off Woods Hole, Mass. This figure clearly indicates the spreading out of the current toward the right, which is necessary to prevent instability, whereas the left side of the current shows the packing of the jet.

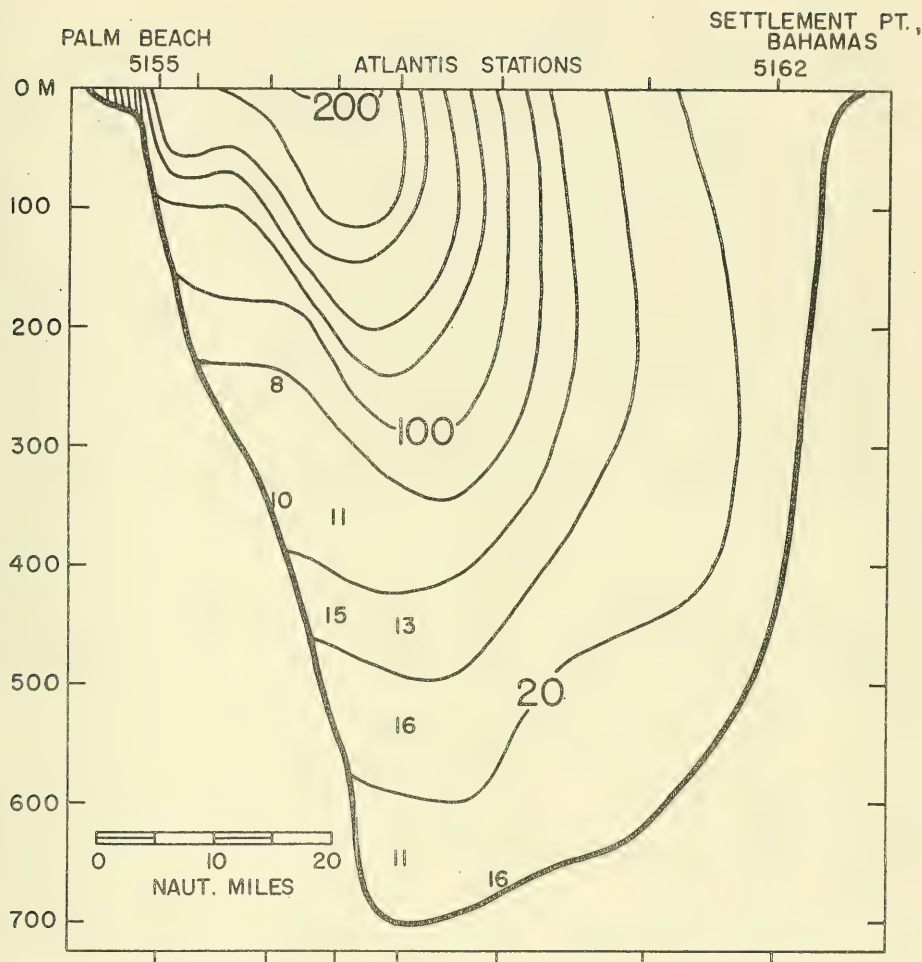


FIGURE 5. VELOCITY SECTION (cm./sec.) ACROSS THE FLÓRIDA CURRENT, 7 FEBRUARY 1954. THE SMALL NUMBERS ON THE LEFT-HAND SIDE OF THE CURRENT ARE SALINITY ANOMALIES. (EXPLANATION IN TEXT).

Figure 6 is taken from Von Arx, Bumpus, and Richardson (1954). In this figure the velocities of the Gulf Stream measured just three weeks previous to Operation STANDSTILL are shown by arrows. Three eddies are delineated by the clockwise turning of the vectors at distances of 30 to 50 miles away from the jet and on the right-hand side of the flow. It is evident from the arrows on 29 May that previous to the formation of an eddy on 31 May the vectors had a northward component, and the flow must then have been stable. The data do not allow determination of the exact period of formation of the eddies because of the sporadic nature of the observations. The above development leads to approximately the right period but has a serious flaw in that the radius of the eddy must be small because the entire mass of water is in motion. Inasmuch as the observed currents in the vicinity of the anchor station averaged less than 0.5 knots and were not regular in direction, any existing eddies must have had a radius of less than 2 miles. However, the radius on which the ship swung at anchor (disregarding drag on the bottom) was a minimum of 2.5 miles, because 20,000 feet of anchor cable were payed out in a depth of 900 fathoms. It is therefore evident that the phenomenon which caused the regular change in isothermal depth shown by the spectral analysis regardless of the location of the ship on its anchor arc was much larger in diameter than the small instability eddies.

B. Inertia Waves

A second form of motion that may be considered is inertia waves controlled by the Coriolis force. The equations of motion for these waves considering a finite depth h and postulating that the vertical velocity w is equal to zero both at depth h and at the surface ($z = 0$), may be written as

$$\begin{aligned}\frac{\partial u}{\partial t} + w \cdot 2\omega \cos \phi - f v + \alpha \frac{\partial p}{\partial x} &= 0, \\ \frac{\partial v}{\partial t} + f u &= 0 \\ \frac{\partial w}{\partial t} - 2\omega u \cos \phi + \alpha \frac{\partial p}{\partial z} &= 0\end{aligned}\tag{7}$$

where $\alpha = \frac{1}{\rho}$ is the specific volume. These equations are similar to those on p. 289 of Haurwitz (1941) except that in the present case, the inertial motion is given for all latitudes instead of only at the poles. Equations (7) do not contain a gravity term because the water is considered to be homogeneous and the motion thus entirely inertial.

The equation of continuity completes the initial set of conditions:

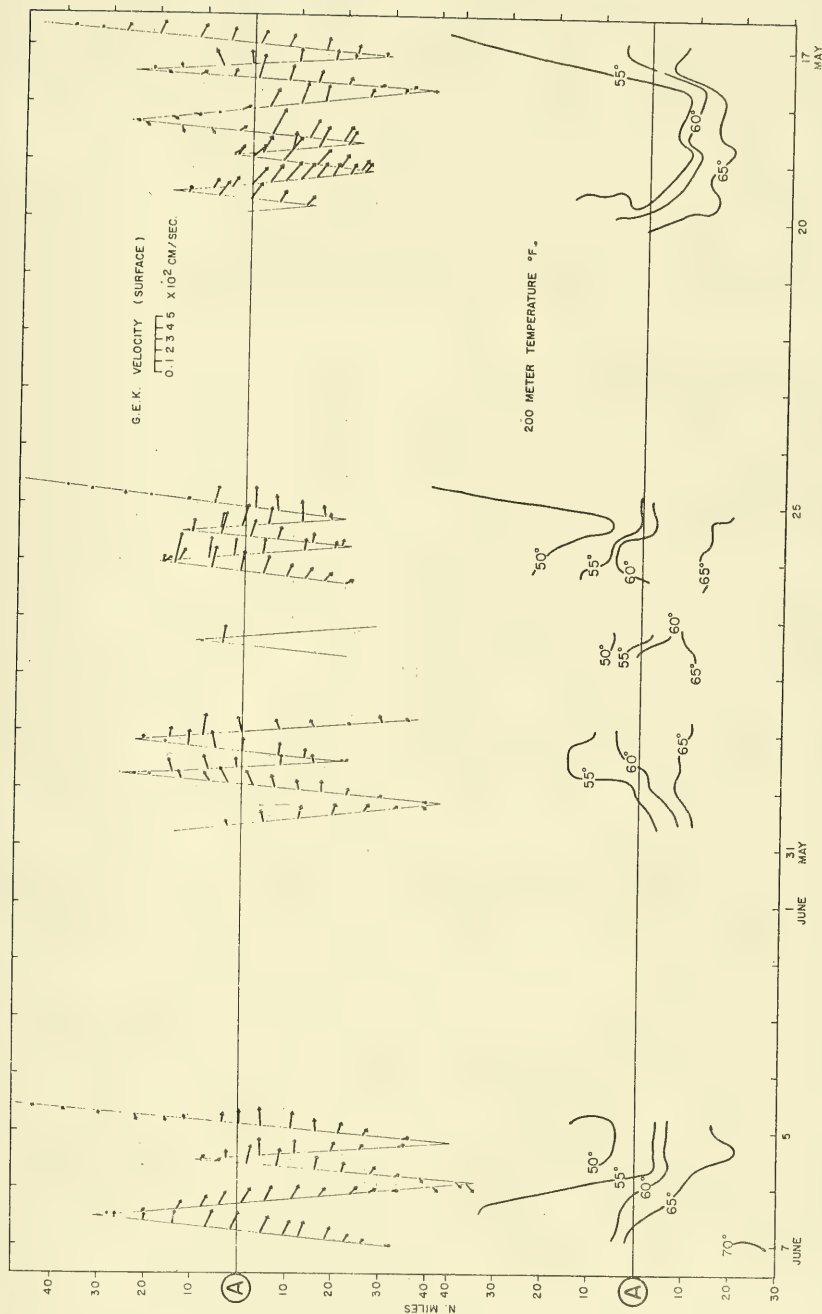


FIGURE 6. VELOCITY VECTORS AND 200-METER ISOTHERMS IN THE GULF STREAM OFF WOODS HOLE, MASS., MAY - JUNE 1953

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (8)$$

for east-west motion.

Considering a free oscillation of inertial type, the wave particle velocities in the x , y , and z directions are:

$$u = e^{i(kx + \sigma t)} \left(A \cos \frac{n\pi z}{h} + B \sin \frac{n\pi z}{h} \right) \quad (9)$$

$$v = e^{i(kx + \sigma t)} \left(C \cos \frac{n\pi z}{h} + D \sin \frac{n\pi z}{h} \right) \quad (10)$$

$$w = E e^{i(kx + \sigma t)} \sin \frac{n\pi z}{h} \quad (11)$$

where $\sigma = \frac{1}{T}$ is the frequency.

To these must be added the driving force

$$p = e^{i(kx + \sigma t)} \left(F \cos \frac{n\pi z}{h} + G \sin \frac{n\pi z}{h} \right). \quad (12)$$

The solution of equations 9, 10, 11, and 12 for the unknowns A through G is accomplished by substitution in (7) and by the combination of terms to separate the coefficients of the sine and cosine factors. Since the sine and cosine are linearly independent, their sum must be zero. Hence,

$$(i\sigma A - fC + i\alpha kF) \cos \frac{n\pi z}{h} + (2\omega \cos \phi \cdot E + i\sigma B - fD + i\alpha kG) \sin \frac{n\pi z}{h} = 0 \quad (13)$$

$$(fA + i\sigma C) \cos \frac{n\pi z}{h} + (fB + i\sigma D) \sin \frac{n\pi z}{h} = 0 \quad (14)$$

$$\left(\frac{\alpha n\pi}{h} G - 2\omega \cos \phi \cdot A \right) \cos \frac{n\pi z}{h} - \left(\frac{\alpha n\pi}{h} F + 2\omega \cos \phi \cdot B - i\sigma E \right) \sin \frac{n\pi z}{h} = 0, \text{ and} \quad (15)$$

$$(ikA + \frac{n\pi}{h} E) \cos \frac{n\pi z}{h} + ikB \sin \frac{n\pi z}{h} = 0 \quad (16)$$

Therefore, the following equations must be satisfied:

$$fC - i\sigma A - i\alpha k F = 0 \quad (17)$$

$$fD - i\sigma B - 2\omega \cos \phi \cdot E - i\alpha k G = 0 \quad (18)$$

$$fA + i\sigma C = 0 \quad (19)$$

$$fB + i\sigma D = 0 \quad (20)$$

$$2\omega \cos \phi \cdot A - \alpha \frac{n\pi}{h} G = 0 \quad (21)$$

$$2\omega \cos \phi \cdot B - i\sigma E + \alpha \frac{n\pi}{h} F = 0 \quad (22)$$

$$ikA + \frac{n\pi}{h} E = 0, \text{ and} \quad (23)$$

$$ikB = 0. \quad (24)$$

From (24) and (20), it is evident that

$$B = D = 0.$$

Solving (19), (21), (22), and (23) in terms of A gives

$$E = -i \frac{kh}{n\pi} A \quad (25)$$

$$C = i \frac{f}{\sigma} A \quad (26)$$

$$G = \frac{2\omega h}{\alpha n\pi} \cos \phi \cdot A, \text{ and} \quad (27)$$

$$F = -\left(\frac{\sigma^2 - f^2}{2\sigma k}\right) A. \quad (28)$$

Substituting (23) and (21) in (18) gives the identity

$$2 \omega \cos \phi \cdot \frac{ik h}{h \pi} - ik \frac{2 \omega h \cos \phi}{h \pi} = 0$$

proving the consistency of the solution. Next, substituting (23) and (17) into (22) gives

$$\frac{-\sigma k h}{h \pi} + \frac{\alpha n \pi}{h} \cdot \frac{\sigma^2 f^2}{\alpha \sigma k} = 0. \quad (29)$$

Hence,

$$\sigma^2 k^2 h^2 = n^2 \pi^2 (f^2 - \sigma^2),$$

$$\sigma = \frac{f}{\left[1 + \left(\frac{2h/L}{n}\right)^2\right]^{\frac{1}{2}}} \quad (30)$$

since $k = \frac{2\pi}{L}$

In shallow water, i.e., when $h \ll L$, $\sigma \approx f$, and it is seen that the period would be approximately the same as in the case of instability eddies, or 21.7 hours. However, since h/L is positive, even though small, the denominator in (30) is greater than 1, and hence $\sigma = \frac{f}{\dots}$ is less than f . The period is then somewhat greater than 21.7 hours.

The physical effect of the inertial motion described above may be seen by considering the motion in the horizontal (xy) plane. In this case, for a fixed z , equations (9) through (12) reduce to

$$u = \cos(kx + \sigma t) \cdot A \cos \frac{n\pi z}{h}$$

$$v = \sin(kx + \sigma t) \cdot C \cos \frac{n\pi z}{h} \quad (31)$$

$$p = -\sin(kx + \sigma t) \cdot H(z).$$

The pressure gradient in the x direction is

$$\frac{\partial p}{\partial x} = -k H(z) \cos(kx + \omega t) . \quad (32)$$

Hence,

$$\frac{u^2}{A^2 \cos^2 \frac{n\pi z}{h}} + \frac{v^2}{C^2 \cos^2 \frac{n\pi z}{h}} = 1 . \quad (33)$$

Thus, the hodograph with respect to time is in the form of an ellipse, and also the particle motion is elliptical in the xy plane. The horizontal particle path is shown in figure 7. The horizontal and vertical motions are represented schematically in figure 7. The figure consists of three parts. The uppermost represents the ellipses of motion near the surface and near the bottom, which are 180° out of phase. The middle portion of the figure shows an east-west vertical section with one wavelength represented. At the points marked with plus and minus signs the flow is out of and into the paper respectively. Three pressure surfaces, P_1 , P_2 , and P_3 , indicate the perturbation pressure gradients. It will be noted that P_2 is shown somewhat nearer the bottom than the corresponding depth $h/2$, and that for this pressure there is no horizontal pressure gradient and thus no horizontal motion; all the motion is vertical at this point. This follows since the second term in equation (12) is not zero at $h/2$, so that the minimum perturbation pressure $P = 0$ is found at some $z = h/2$ plus a small amount. The third part of the figure shows the displacement of the 66° F. isotherm in relation to the wave propagation. In actuality, the wavelength is essentially that of the earth's circumference, so that only a portion of one cell is found in the Atlantic Ocean. However, the complete cycle passes each point in the ocean during a lunar day as the wave propagates westward.

Figure 3 may be reexamined in the light of the above findings. It is suggested that since the inertial period is 21.7 hours plus an unknown amount depending on the ratio of the depth to the wave length (from equation 30), the spectral bands centered at 20 and 24 hours show the greatest power. There is a sharper peak in the band from 21.8 to 26.7 hours possibly because the diurnal tide acts in resonance with the inertial force. Correspondingly, it is possible that no semidiurnal period shows on the power spectrum because the inertial filter is "transparent" to waves of 12-hour period; i.e., the natural period (21.7 hrs.) of the inertial oscillations is unresponsive to the 12-hour tidal constituent.

In the case of stratified water, where a component due to gravity modifies σ^2 by analysis similar to that given above, it can be shown that σ for the free wave is given by the following approximate relation:

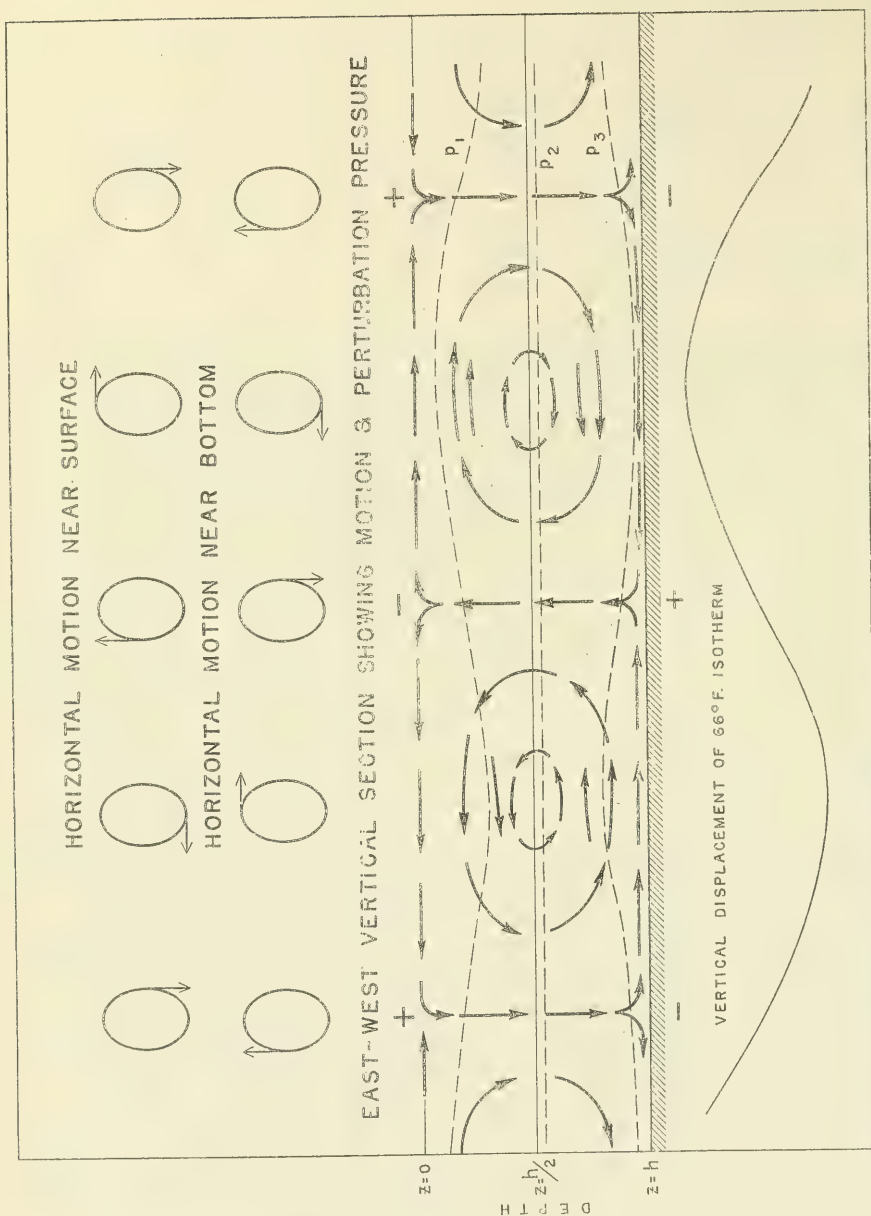


FIGURE 7. GRAPHICAL REPRESENTATION OF HORIZONTAL AND VERTICAL MOTION AND DISPLACEMENT OF 66°F. ISOTHERM FOR WAVE PROPAGATING WESTWARD

$$\sigma^2 = \frac{h^2 k^2 g (\frac{1}{\rho} \frac{\partial \rho}{\partial z})}{h^2 \pi^2 + h^2 k^2} + \frac{f^2}{1 + (\frac{hk}{h\pi})^2} \quad (34)$$

where $\frac{1}{\rho} \frac{\partial \rho}{\partial z} \sim$ (stability due to stratification)

VI. SUMMARY AND CONCLUSIONS

An analysis for the spectral periods of internal waves was made from temperature readings, comprising nearly 1,200 bathythermograms taken at half-hourly intervals during Operation STANDSTILL. The analysis shows no significant power associated with periods from 1 to 20 hours, but a concentration of the power in the band of periods between 20 and 26 hours. If there is no aliasing from periods of less than 1 hour, it may be concluded that the analysis of the temperature data fails to show that internal waves of periods of less than 20 hours carry any appreciable amount of power at the location of the operation.

Two possible causes are suggested for the predominance of spectral periods between 20 and 26 hours: formation of eddies on the southern edge of the Gulf Stream due to instability gives periods of 21.7 hours, but the diameter of the eddies is too small to account for the observed isothermal changes; internal motion due to forced inertial oscillation is suggested, with a period somewhat more than 21.7 hours. The latter oscillation apparently is in resonance with the diurnal tidal components, and as a result the spectral power is concentrated in the observed periods.

*See Fjeldstad, J. E., Interne Wellen, Geofysiske Publikasjoner, Vol. X, No. 6, Oslo, 1933.

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